

# The Demand for Immigrants in an Overlapping Generations Economy

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## ABSTRACT

This paper develops a novel framework to analyze the dynamic interaction of capital accumulation, population growth, and immigration policy. This framework combines two different features which seem to prevail in several countries: a decentralized behavior of the agents in factors and goods markets, and a coalitional attitude with respect to the immigration process. Our model considers an overlapping generations economy in which agents make lump-sum contributions to an “immigration agency,” which uses its proceeds to finance a program of immigration subsidies. The agency seeks to maximize the lifetime utility of the contemporaneous agents, taking into account the posterior decentralized reactions of the individuals to its own decisions. The model is then used to study, through tentative simulations, the welfare implications of a policy of immigrants-subsidy in the United States. It is found that the long-run effect is nearly nil, because the immediate benefits from immigrants are partially counteracted by a negative effect on capital accumulation.

KEYWORDS : Immigration; Decentralized Equilibrium; Welfare.

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## Introduction

Under standard assumptions, economic theory prescribes that free mobility of labor represents a source of potential economic gains. Moreover, recent empirical studies suggest that the benefit to the world economy from removing barriers on productive factors might be significant, and might even exceed the benefit from removing restrictions on goods trade.<sup>1</sup> The logic behind such a claim should be apparent: given differences in endowments, residents of capital-intensive countries could improve by employing immigrant labor, and residents of labor-intensive countries could improve by offering their labor abroad. It seems quite natural to conclude that the typical immigration policy should not impose too many obstacles to the entry of potential immigrants.

In practice, however, different groups inside the host economy might be unequally affected by the immigration process: that is, some groups might be benefited by the entry of new hordes of immigrants, while others might be harmed, either in a short or a long-run perspective. Indeed, the economic prescription in favor of free flow of labor implicitly presumes the existence of redistribution mechanisms, so that the losses of some groups are compensated by the greater gains of some others. Since those mechanisms are usually hard to implement—specially if the cost and benefits of the process are spread over long periods of time—the immigration policy is a matter of political economy, and it depends on the relative strengths of different forces within the host economy.<sup>2</sup>

The appropriate analysis of these issues requires the explicit introduction of some notion of heterogeneity among the set of economic players. The most common approach in that direction relies on a static separation of the agents in accordance with endowments or income sources. Not surprisingly, it is then found that capital owners should be favorable to immigration, while labor organizations should oppose it. Although this framework is perhaps convenient for short run analysis, it can hardly be useful to describe an evolutive economy. For one thing, in a dynamic economy people accumulate capital over time, and the division between capitalists and workers is not conceptually straightforward. Moreover, a relevant problem in the study of immigration processes is the contribution of immigrants to the accumulation of capital inside the host economy, an important feature which cannot be appropriately addressed in a static capital-labor environment.<sup>3</sup>

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<sup>1</sup> Hamilton and Whalley (1984) estimated that global income would double in the absence of immigration restrictions, and conclude that the immigration issue was “one of the (and, perhaps, the) most important issues facing the global economy.”

<sup>2</sup> In that sense, Grether et al. (2000) report that polls conducted in US and Europe show that a significant majority of those interviewed believe that ‘immigrants are too many’. These authors then conclude that “on economic ground, at least, one might be inclined to expect a more positive attitude towards immigration than those expressed in recent attitude surveys.”

<sup>3</sup> A case in point is the discussion concerning the social security in several OECD countries, where the solvency of the system in the near future might require a significant increase in the working population. For instance, population projections based on current trends suggest that the flow of immigrants in the EU as a whole should reach 13.5 million a year to keep

On the other hand, an overlapping generations framework seems to be a very natural tool: it provides a simple world in which different types of agents interact with each other over time. In that simple world, the economic problem of individual agents could be easily posed and frequently analytically solved, so that the resulting evolution of capital and population could be examined in a tractable way. Following that intuition, this paper presents an overlapping generations model to study the dynamics of an economy with immigration.

Our approach distinguishes itself by several novelties. A main contribution is to combine two issues which seem to be observable in several countries —a decentralized behavior of the individual agents in the factors and goods markets, and a sort of collectively agreed-upon attitude with respect to immigration. The underlying rationale for the latter feature is the basic perception that economic agents similarly affected by the arrival of immigrants have an incentive to gather into coalitions, and that such coalitions are important ingredients for the design of the immigration policy of a country. Therefore, the paper offers a theoretical framework to analyze the economic factors underlying immigration policies — namely, the costs and benefits of immigration, their distribution among different agents, and its evolution over time.

A second contribution is to explore the dynamic link between the state of the host economy, as reflected in wages and interest rates, and the demand for immigrants. In that sense, our model extends to a long run perspective the commonly held idea that the attitude with respect to immigration is highly depending on the phases of the business cycle.<sup>4</sup> And, finally, we also incorporate into the analysis the fact that, as immigrants settle down in the host economy, their own preferences eventually begin to influence the prevailing immigration policy. The recognition of this fact is a common place in informal discussion of immigration, but its theoretical formalization has turned out to be somehow elusive. Therefore, our treatment of that aspect represents a contribution to the previous literature.<sup>5</sup>

The structure of this paper is as follows. In section 2, the most relevant literature is briefly surveyed. In section 3, we set up the basic structure of the model, and in section 4 we analyze the welfare implications of a policy of subsidizing immigration. In particular, it is shown that the promotion of the immigration process by part of a given generation increases the welfare of its members, but it might also reduce the steady-state level of capital and utility. In

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a steady ratio of pensioners to workers. Inasmuch as the survival of the system is mostly convenient for current workers, those should be *in favor* of a greater number of immigrants, rather than oppose them as the static model would predict. This highlights the limitations of such a framework for the analysis of the political economy of immigration. See Drinkwater et al. (2002), pg. 26 for further discussion on this issue.

<sup>4</sup> See, for instance, Gador and Stark (1990), p. 463. In turn, Borjas (1996) has argued that “simple economics and common sense suggest that the magic number (of admitted immigrants) should not be an immutable constant regardless of economic conditions in the United States.”

<sup>5</sup> For example, in a related vein, Myers and Papageorgiou (1997) emphasize that if current trends continue, the recently immigrant groups “could acquire economic and political strength sufficiently to implement deep changes in the overall way the historical mainstream of a country organizes and conducts its affairs.” However, see Chang (1996) for a discussion of the difficulties of incorporating this fact into the analysis.

section 5, we extend the model to consider a two-country game in which migrants can move in either direction, depending on the policies of both countries. The analytical discussion of the model is followed in section 6 by a simulation which roughly replicates the dynamics of capital accumulation, population growth and immigration policy in the United States. The results suggest that the long-run welfare effect of a policy of immigration-subsidy are nearly nil, because its short-run benefits are offset by a decrease in the long-run level of capital. The paper is closed with its main conclusions and suggestions for further research.

## 1 Background

In the recent past, several countries have experienced a remarkable inflow of immigrants. By the late 1980's, it was estimated that over 60 million people—i.e., around 1.2 percent of world population—resided in a country where they were not born. More recent estimations suggest that around 6% of the population in France, 19% in Switzerland, 9% in Belgium, 4% in the United Kingdom, and 8% in the United States is foreign-born.

Although a considerable part of the huge immigration process obeys to political reasons, it is widely believed that economic impulses are even stronger. Thus, the rise of population mobility has been followed by a significant wave of economic research. Most of the contributions have been empirical, and the main general questions might be summarized as follows: (a) what is the impact of immigrants on the employment opportunities of natives?, (b) how do immigrants perform as compared with native-born population?, and (c) what is the net contribution of immigrants to the welfare state? Borjas (1999), Commander et al. (2002), and Drinkwater et al. (2002) present update surveys of the literature on those issues, and an interesting historical perspectives can be found in Goldin (1994) and Foreman-Peck (1992).

The theoretical analysis of immigration has not evolved with the same strength. Thus, two decades ago it was recognized that migration was a central feature of the international economy, but “it has never received more than a small fraction of the attention lavished on the theory of international capital movement.”<sup>6</sup> However, while the theoretical study of immigration is still noticeably narrow, more recent developments have corrected in part such unpleasant neglect, and we will highlight those closest to our work. The earliest relevant work can be traced to Ethier (1985, 1986) and Bhagwati and Srinivasan (1983). The first two papers addressed the effects of alternative immigration policies on the level of employment and national income, and the third one compared the welfare implications of restrictions on capital and labor mobility.<sup>7</sup> Those models were essentially static and, in particular, the process of capital accumulation was ignored. Moreover, the analyses relied on exogenous demand functions which

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<sup>6</sup> Ethier (1985).

<sup>7</sup> A relevant antecedent for the general investigation of international migration is the classic paper by Harris and Todaro (1970), where the aggregate effects of migration from rural into urban areas of underdeveloped countries was first examined.

were not explicitly linked to the utility-maximizing behavior of economic agents. These two features limit the relevance of those models for long run dynamics. The first consideration also applies to a static model developed by Myers and Papageorgiou (1997), in which an active governmental control of immigration is considered.

The model in Djajic and Milbourne (1988) partially overcomes the limitations in the preceding work. In contrast with earlier studies, it is based on the notion that migrants are utility-maximizing individuals with finite working lives, and the asset accumulation of a typical worker is viewed as part of a solution to an intertemporal optimization problem. Those ingredients are mixed to study the general equilibrium of the source-economy, but the effects of immigration on the host-economy is not approached. Braun (1993) introduced immigration into a standard neoclassical growth model, and further variations of his models are presented in Barro and Sala-i-Martin (1995). Since those models evolve in the context of a representative-agent economy, the distributive effects of immigration cannot be possibly considered. Moreover, it is a well-known (and intuitive) fact that, in such a context, the welfare of the native population must necessarily improve with the arrival of immigrants, and then the discussion of welfare issues in that world happens to be uninteresting.<sup>8</sup> Steineck (1996), however, shows different models in which immigrants might have negative effects on native welfare because they slow down the rate of technological progress and economic growth.

The theoretical approach of immigration is particularly in need of further work on the political economy aspects of the immigration process. In that direction, Benhabib (1996) studies how immigration policies that impose capital and skill requirements would be determined under majority voting when natives differ in their wealth holding and vote to maximize their income. Chang (1996) offers a rich, although less formalized analysis of normative and positive aspects related to distributive effects of immigration. In turn, Ghatak et al. (1996) present an extension of the Harris-Todaro model to analyze the welfare implications of government intervention in the form of employment subsidies. Schiff (1998) highlights the negative impact of immigration on the social capital of the host country, in order to illustrate why free trade might be promoted at the same time that free migration is surprisingly opposed. Other authors, such as Levine et al. (2002) have embedded the immigration process in an endogenous growth framework, in which the immigrants have welfare implications through size effects and human capital dilution. Finally, Sempere (2000) explore the difficulties to design an appropriate redistribution mechanism to obtain a Pareto gain from freeing migration.

The present paper can be related to some additional work. Gador (1986) has studied the welfare implications of international immigration in an overlapping generations framework. The notion of “agency,” which we shall introduce later, is related to the “council” used by Kotlikoff, Persson and Svensson (1987) for different purposes. Lastly, John and Pecchenino (1994, fn. 10) informally de-

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<sup>8</sup> See Berry and Soligo (1969).

scribe a problem to some extent similar to ours in the context of environmental economics.

## 2 The Model

### 2.1 The Environment

Consider an infinitely-lived economy where agents live for two periods. At each period, a new group of people is born and a group of young immigrants arrives. A generation is defined as the set of native-born individuals in a given period *and* the immigrants who arrived at the same time. Each agent (either a native born or an immigrant) is endowed with one unit of labor during his first period of life, and he supplies this endowment inelastically to receive a competitive wage. The wage income is used for three different purposes: (i) to finance current consumption, (ii) to provide the savings for retirement, and (iii) to make a compulsory lump-sum contribution to an economic club, hereinafter identified as the “agency.”

Each generation forms its own agency. An agency is a political institution which represents the special interest of contemporaneous individuals. Its economic role is to collect the contributions from its members when they are still young, and to use the proceeds to finance the implementation of a convenient immigration policy. More specifically, the objective of the agency formed at period  $t$  is to select the contribution fee and corresponding immigration policy—defined as a level of effort against or pro immigration—in such a way that the discounted utility of a representative agent of generation  $t$  is maximized. Having implemented its program, the agency formed at time  $t$  exits the model at time  $t + 1$ , and is then replaced by a new agency which represent the special interest of the generation  $(t + 1)$ .

The agency should not be confused with a conventional social planner. The social planner would be long-lived, and it should be concerned with the welfare of all generations; on the other hand, the agency is short-lived, and is only concerned with the welfare of its contemporaneous generation. It would be more natural to think of the agency as a government whose policy is decided by majority vote—assuming the current generation is larger than the previous one—, but is politically constrained in its ability to impose excessive burden on older generations to finance activities which are not beneficial to them. Alternatively, it can be taken as a theoretical abstraction for the self-evident fact that agents with economic affinities have a tendency to form coalitions in defense of their common interest. The economic affinity in our model is given by the age of the agents, which is in turn a surrogate of their wealth.

Each individual agent behaves in a competitive fashion, taking factor prices as given. She also takes the contribution demanded by the agency as an exogenous parameter. The agency, however, recognizes that its actions affect the future path of capital, population, and prices. Thus, it will be assumed that the agency formed at period  $t$  implements a Ramsey policy: it selects the immi-

gration fee and immigration policy so that the welfare of a representative agent of generation  $t$  is maximized, taking into account the posterior behavior of the economy once those parameters have been imposed.

The investment decisions of the agents at a period  $t$  are affected by the expectation of the prices at period  $t + 1$ . On the other hand, the prices at period  $t + 1$  depends on the previous savings decisions of the agents. This yields a typical interaction in which the expectations of future prices affect current decisions, which in turn determine future prices. We will analyze a perfect foresight equilibrium in which those relationships are mutually consistent, in the sense that the expectations of individual agents are fulfilled through their own optimizing decisions.

## 2.2 Household and Firms

Let us assume that preferences are the same for all agents in all generations. Moreover, let us say that the preferences of an agent of generation  $t$  can be represented by an additive and twice-differentiable utility function, described over all nonnegative consumption bundles. That is,

$$\mathcal{U}(c_{1t}, c_{2t+1}) = u(c_{1t}) + \beta u(c_{2t+1}), \quad 0 < \beta < 1 \quad (1)$$

where  $c_{1t}$  denotes his consumption in the first period of life, and  $c_{2t+1}$  denotes his consumption in the second. It is also assumed that  $u' > 0$  and  $u'' < 0$ , so that  $u$  is increasing and strictly concave, and that

$$\begin{aligned} \lim_{c \mapsto 0} u'(c) &= \infty \\ \lim_{c \mapsto \infty} u'(c) &= 0 \end{aligned}$$

Then, the household's problem is

$$\max_{c_{1t}, c_{2t+1}} u(c_{1t}) + \beta u(c_{2t+1}) \quad (2)$$

$$s.t. \quad c_{1t} + \frac{c_{2t+1}}{R_{t+1}} = w_t - \tau_t \quad (3)$$

where  $\{\tau_t, w_t, R_{t+1}\}$  are supposed to be known and exogenously given from the individual agent's perspective.<sup>9</sup> Conventionally,  $w_t$  represents the wage income at time  $t$  and  $R_{t+1}$  represents the (gross) interest rate at period  $t + 1$ . In turn,  $\tau_t$  denotes the contribution of the representative agent of generation  $t$  to the immigration agency.<sup>10</sup> The optimal behavior of the agency will be characterized below.

From the continuity of  $u$  and the compactness of the constraint set, a solution for the consumer problem exists for any finite interest rate. Moreover, from the

<sup>9</sup> Alternatively, one might consider  $R_{t+1}$  as the expected interest rate, and then endow the agents with perfect foresight.

<sup>10</sup> It is also possible to assume that the utility of the households depends on the *received* transfer,  $\tau_{t-1}$ . Since  $\tau_{t-1}$  is taken as given at period  $t$ , that does not represent any important issue.

strict concavity of  $u$  and the convexity of the constraint set, the solution is single-valued. Standard conditions for the interior solution of this problem can be immediately stated as:

$$\frac{u'(c_{1t})}{u'(c_{2t+1})} = \beta R_{t+1} \quad (4)$$

Combining (4) with the budget constraint in (3), we can derive a continuous consumption and savings function

$$c_{1t} = c_{1t}(w_t - \tau_t, R_{t+1}) \quad (5)$$

$$s_t = s_t(w_t - \tau_t, R_{t+1}) \quad (6)$$

Introducing (5) and (6) into the objective, we obtain an indirect utility function  $V(w_t - \tau_t, R_{t+1})$ . From the Maximum Theorem,  $V$  is a continuous function for finite values of  $R_{t+1}$ . It is clearly increasing in  $w_t$  and  $R_{t+1}$ , but decreasing in  $\tau_t$ .

From (4),

$$u'(w_t - \tau_t - s_t) = \beta R_{t+1} u'(R_{t+1} s_t) \quad (7)$$

Assuming that  $s_t$  is differentiable, we obtain the conventional results

$$0 < s_w = -s_\tau = \frac{u''(c_{1t})}{u''(c_{1t}) + \beta R_{t+1}^2 u''(c_{2t+1})} < 1 \quad (8)$$

and

$$s_r \gtrless 0. \quad (9)$$

On the other hand, the firm's problem is:<sup>11</sup>

$$\max_{K_t, P_t} \Pi(K_t, P_t) \equiv F(K_t, P_t) - w_t P_t - r_t K_t$$

where  $K_t$  represents the amount of physical capital,  $P_t$  denotes the working population, and  $r_t$  is the (net) interest rate at period  $t$ .  $F$  is a standard neoclassical production function, so that the usual Inada conditions are assumed to hold.

For convenience, we ignore depreciation and technical progress. Hence, under perfect competition, the absence of arbitrage opportunities leads to

$$R_t = F_k(K_t, P_t) + 1, \quad (10)$$

and

$$w_t = F_p(K_t, P_t) \quad (11)$$

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<sup>11</sup> Under constant returns to scale, the assumption of a single firm is unarmful. The results are the same as with many identical competitive firms.

where subscripts  $k$  and  $p$  denote partial derivatives.

Let us say that  $F(.,.)$  has constant returns to scale, so that we obtain:

$$w_t = \omega(k_t) \equiv f(k_t) - k_t f'(k_t) \quad (12)$$

$$R_t = r(k_t) \equiv f'(k_t) + 1 \quad (13)$$

where  $k_t \equiv \frac{K_t}{P_t}$  and  $f(k) \equiv F(k, 1)$ . Of course,  $\omega'(k_t) = -k_t f''(k_t) > 0$ , while  $r'(k_t) < 0$ .

### 2.3 The Immigration Function

Let  $M_t$  be the number of new immigrants arriving at period  $t$ . It will be postulated that

$$\frac{M_{t+1}}{P_t} = \Omega(w_{t+1}, \tilde{w}_{t+1}) + \phi(e_t), \quad \Omega_1 > 0, \phi' > 0 \quad (14)$$

where  $\tilde{w}_{t+1}$  denotes the exogenous opportunity cost of immigrants, and  $e_t \gtrless 0$  denotes the average effort (among  $t$ -generation's agents) against or pro immigration. It is clear that  $e_t$  satisfies:

$$|e_t| = \tau_t \leq w_t. \quad (15)$$

The intuition behind the expression in (14) should be quite appealing. Its first term is similar to the expression used by Barro and Sala-i-Martin (1995). The effect of wage differential between the source and destination countries is also emphasized in Djajic and Milbourne (1988) and Gador and Stark (1990). For convenience, and without loss of generality, we shall further assume that  $\{\tilde{w}_t\}$  is constant over time, and we shall normalize it to zero. A sufficient condition for that feature is that the supply of workers from the rest of the world is unlimited, and then foreign wages are not affected by level of immigration  $M_{t+1}$ . Other alternatives for the specification of the function are of course possible. For example, it might be desirable to introduce the number of previous immigrants as an argument of the function  $\Omega$ , which is the approach in Guzmán (1997). Similarly, individual's decision to emigrate might be affected by the prospective return on savings, rather than solely by the current wage, so that the inclusion of  $R_{t+2}$  as an additional argument might be appropriate.<sup>12</sup> In this paper, we will proceed in a simplified fashion, postponing those complicating features until later extensions.

<sup>12</sup> A technical reason to exclude the interest rate from  $\Omega$  is that its inclusion would lead to a third order difference equation for the dynamics of capital, which is hard to characterize analytically. Economically, one might argue that prospective immigrants do not have enough information about the future path of the foreign economy, so that the influence of future interest rates on their decisions is weak. In any case, the inclusion of the feature is not qualitatively important: it basically increases the incentive of prospective immigrants to move into the host economy, reducing the optimal amount of subsidy from the agencies. An empirical exploration of the model might easily incorporate that effect.

The function  $\phi$  symbolizes the ability of the immigration agency to control the immigration flow through its policy. It simply formalizes the common sense observation that international migration depends not only on the decisions of prospective migrants, but also on the policies of the host country.<sup>13</sup> In turn, the variable  $e_t$  is an indicator of the direction and intensity of the “immigration policy” adopted at period  $t$ . A positive  $e_t$  might be interpreted as describing the construction of hospitals, schools, public services, and facilities primarily enjoyed by immigrants, as well as the use of resources for social programs to which many immigrants are entitled. A negative  $e_t$  might be thought as the average cost of police training, detection technology and infrastructure aimed to control immigration, or the average cost of economic resources given to other countries to improve their economic conditions and reduce the incentive of their citizens to emigrate.<sup>14</sup> It is important to point out that the effect of a particular level of  $e_t$  operates at period  $t + 1$ , so that the old generation a any given period is not affected by the current immigration policy of the current agency, as implied by the political constraint of the  $t$ -agency to impose tax burden on the old generation.

For simplicity, it will be assumed that the number of new born at any period is equal to the number of agents getting old. Hence, without immigration, the total working population in this economy would remain constant.<sup>15</sup> Given the arrival of immigrants, we have instead

$$P_{t+1} = P_t + M_{t+1}. \quad (16)$$

It follows immediately that

$$P_{t+1} = P_t(1 + \phi(e_t) + \Omega(w_{t+1})), \quad (17)$$

or, equivalently,

$$\frac{P_{t+1}}{P_t} = 1 + n_t, \quad (18)$$

where

$$n_t \equiv \phi(e_t) + \Omega(w_{t+1}) \quad (19)$$

In other words, the population grows at the variable rate  $n_t$ , where  $n_t$  depends on the level of wages, and on the *per capita* level of effort against or pro immigration.

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<sup>13</sup> Gador and Stark (1990) also remark this point, but they do not dwell on the idea.

<sup>14</sup> In that respect, two relevant examples are the transfer of financial resources to Mexico during the peso crisis and the US expenditures seeking to improve the economic conditions of Haiti. Both efforts can be (and, indeed, they were) understood as efforts of the United States to control the immigration flow from those countries by improving their internal economic conditions.

<sup>15</sup> The introduction of a positive natural rate of population growth is straightforward, but it is not relevant for our purposes.

## 2.4 The Agency Problem

The problem of the agency formed at  $t$  is to select a level of effort and “taxation” such that the lifetime utility of a typical  $t$ -generation agent is maximized. To this end, the agency follows a Ramsey policy; namely, it maximizes its objective subject to the demand and supply functions of individuals, which are in turn the solution of a maximization problem in which agency’s decisions are taken as constraints. Once again, it might be convenient to emphasize that “generation  $t$ ” also includes those immigrants who arrive in that period. Thus, we are explicitly assuming that the welfare of a new immigrant is taken into account for the contemporaneous agency.<sup>16</sup> Again, it is assumed that the agency is not able to tax the old generation, so that it is financed by the people for whom its policy is beneficial. The agency’s problem is

$$\begin{aligned}
 \max_{e_t, \tau_t} \quad & V(w_t - \tau_t, R_{t+1}) \\
 \text{s.t.} \quad & (i) \quad \tau_t = |e_t| \leq w_t \\
 & (ii) \quad w_t = f(k_t) - k_t f'(k_t) \\
 & (iii) \quad R_{t+1} = f'(k_{t+1}) + 1 \\
 & (iv) \quad k_{t+1} = \frac{s(w_t - \tau_t, R_{t+1})}{1 + \phi(e_t) + \Omega(w_{t+1})} \\
 & (v) \quad w_{t+1} = f(k_{t+1}) - k_{t+1} f'(k_{t+1}) \\
 & (vi) \quad k_t \text{ given}
 \end{aligned} \tag{20}$$

The constraint (iv) in this problem follows from the equilibrium condition  $K_{t+1} = S_t$ , where capital letters represent aggregate values. That is, by definition,

$$\begin{aligned}
 k_{t+1} &= \frac{S_t}{P_{t+1}} \\
 &= \frac{P_t s(w_t - \tau_t, R_{t+1})}{P_t(1 + \phi(e_t) + \Omega(w_{t+1}))}
 \end{aligned}$$

It will be helpful to simplify the constraints in (i) – (v) in the following way. First, using the equilibrium conditions for  $w_{t+1}$  and  $w_t$ , we readily obtain:

$$k_{t+1} = \frac{s(\omega(k_t) - \tau_t, R_{t+1})}{1 + \phi(e_t) + \Omega(\omega(k_{t+1}))} \tag{21}$$

which can be written as

$$k_{t+1} = h(k_t, e_t, \tau_t, R_{t+1}) \tag{22}$$

Now, from the constraint in (iii), it follows that:

$$\begin{aligned}
 R_{t+1} &= f'(k_{t+1}) + 1 \\
 &= f'(h(k_t, e_t, \tau_t, R_{t+1})) + 1
 \end{aligned} \tag{23}$$

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<sup>16</sup> It is also possible to assume that agents live for several periods, and that an immigrant only becomes socially influential *several* periods after her arrival.

Solving this system, we obtain the difference equations:

$$k_{t+1} = g^1(k_t, e_t, \tau_t) \quad (24)$$

$$R_{t+1} = g^2(k_t, e_t, \tau_t) \quad (25)$$

The following result can now be proved. It is a dynamic rationalization of the common opinion that immigrants should be attracted (or, at least, not be rejected) by the host economy.

**Proposition 1** *Assume the regularity condition  $s_r \geq 0$  holds, and let  $(\tau_t, e_t)$  be the optimal immigration policy at period  $t$ . Then, in the economy described here,  $e_t \geq 0$  for all  $t$ .*

**Proof.** Suppose, by way of contradiction, that  $e_t < 0$  for some  $t$ , and consider the alternative policy  $(\tau_t^*, e_t^*)$  defined as:

$$\begin{aligned} \tau_t^* &= \tau_t \\ e_t^* &= -e_t. \end{aligned}$$

In other words, the agency keeps the current fee constant, but uses the proceeds in favor of immigration, rather than against it. Now, for a given  $k_t$ , let us write (21) as

$$k_{t+1}(1 + \phi(e_t) + \Omega(f(k_{t+1}) - k_{t+1}f'(k_{t+1}))) = s(\omega(k_t) - \tau_t, f'(k_{t+1}) + 1)$$

Differentiating both sides, we get:

$$\frac{dk_{t+1}}{de_t} = -\frac{\phi'(e_t) k_{t+1}}{1 + n_t - (k_{t+1}^2 \Omega' + s_r) f''} \leq 0$$

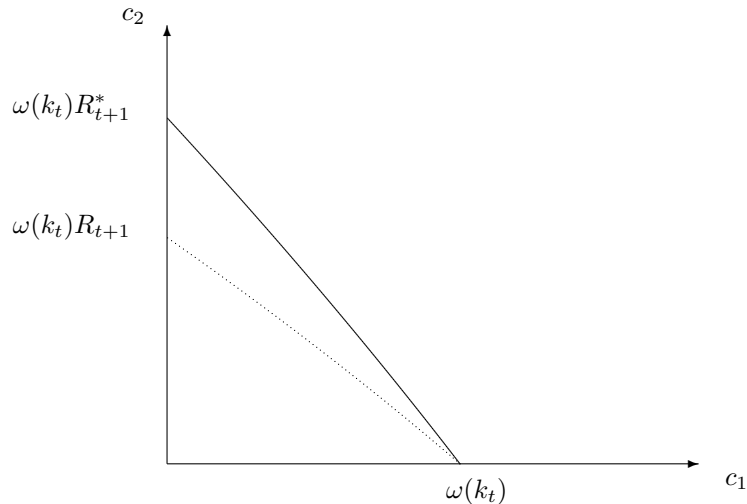
From (23)

$$\frac{dR_{t+1}}{de_t} = f''(k_{t+1}) \frac{dk_{t+1}}{de_t} \geq 0$$

Then, the change of policy leads to a situation in which the current disposable income is unchanged, but  $R_{t+1}$  increases. Since this expands the set of consumption possibilities of the typical  $t$ -generation agent, the original policy cannot be optimal. This is illustrated in Figure 1. ■

The intuition of the proposition above is immediate. Since  $e_t^* > e_t$ , the population  $P_{t+1}$  tends to *increase* for any original  $k_{t+1}$ . Hence,  $k_{t+1}$  would tend to *decrease* with respect to its original value, unless the increase on population is countervailed by an increase on saving. However, with the disposable income held constant, such an increase on saving can only occur with a decrease of  $k_{t+1}$  in the first place. The usefulness of the result is to allow us to set  $\tau_t$  in place of  $e_t$ . We will use that fact to characterize the functions  $g^r$  and  $g^k$ , defined as below.

Figure 1: Consumption Sets under Alternative Policies



An immediate question is whether the path of capital is uniquely determined, for a given immigration policy. This is equivalent to verify that, for any given  $(k_t, \tau_t)$ , there is an unique value of  $k_{t+1}$  satisfying equation (21). If that is not the case, there would be many different equilibrium path for capital accumulation, each one being self-fulfilling, and a perfect foresight equilibrium would not be well defined. The following result settles that issue.

**Proposition 2** *Assume the regularity condition  $s_r \geq 0$  holds, and let  $(k_t, \tau_t)$  be given. Then,  $k_{t+1}$  and  $R_{t+1}$  are uniquely determined.*

**Proof.** We mimic the procedure for the standard OLG model. Thus, in Figure 2, the left and right-hand sides of (21) are depicted as functions of  $k_{t+1}$ , considering  $k_t$  and  $\tau_t$  as given. Since  $\Omega'(w_{t+1}) > 0$  and  $s_t < w_t$  for any pair  $(w_t, k_{t+1})$ , we have that:

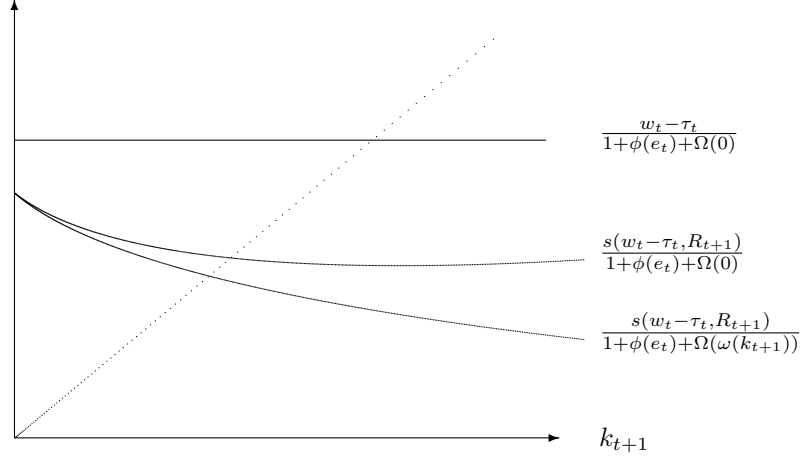
$$\begin{aligned} \lim_{k_{t+1} \rightarrow \infty} \frac{s(w_t - \tau_t, R_{t+1})}{1 + \phi(e_t) + \Omega(w_{t+1})} &< \lim_{k_{t+1} \rightarrow \infty} \frac{s(w_t - \tau_t, R_{t+1})}{1 + \phi(e_t) + \Omega(0)} \\ &< \frac{w_t - \tau_t}{1 + \phi(e_t) + \Omega(0)} \end{aligned} \quad (26)$$

where, for notational convenience, we ignore the arguments of some functions. Then, the existence of a value  $k_{t+1}$  satisfying (21) would be proved if  $\lim_{k \rightarrow 0} s(w - \tau, k) \geq 0$ , which is indeed implied by the assumption  $s_r \geq 0$ .

To prove uniqueness, it suffices to show that the derivative of the RHS with respect to  $k_{t+1}$  is everywhere smaller than 1. Now, it is obvious that

$$(1 + \phi(\tau) + \Omega) s_r + s \Omega' k > 0 \geq \frac{(1 + \phi(\tau) + \Omega)^2}{f''} \quad (27)$$

Figure 2: Determination of  $k_{t+1}$



Since  $f'' < 0$ , this could be rearranged to get:

$$\frac{(1 + \phi(\tau) + \Omega) s_r f'' + s \Omega' k f''(k)}{(1 + \phi(\tau) + \Omega)^2} - 1 \leq 0 \quad (28)$$

The first term is precisely the derivative of the RHS of (21) with respect to  $k_{t+1}$ . Then, the result follows immediately. ■

**Lemma 1** Let  $\tau_t \equiv e_t$ , and let

$$k_{t+1} = g^k(k_t, \tau_t) \quad (29)$$

$$R_{t+1} = g^r(k_t, \tau_t) \quad (30)$$

be the functions implied by the system (24) – (25). Then, under the regularity condition  $s_r \geq 0$ , the following holds:

- (i)  $\partial g^r / \partial \tau_t = \frac{-(s_w + \phi'(\tau_t) + k_{t+1}) f''(k_{t+1})}{\Delta - f''(k_{t+1}) s_r} \geq 0$
- (ii)  $\partial g^k / \partial \tau_t \leq 0$
- (iii)  $\partial g^r / \partial k_t = \frac{-s_w k_t f''(k_t) f''(k_{t+1})}{\Delta - f''(k_{t+1}) s_r} \leq 0$
- (iv)  $\partial g^k / \partial k_t \geq 0$

where

$$\Delta \equiv 1 + n_t - k_{t+1}^2 \Omega'(k_{t+1}) f''(k_{t+1}) \geq 0$$

**Proof.** For any function  $g(x, \cdot)$ , define  $g_x \equiv \frac{\partial g}{\partial x}$ . Differentiation of (21) with respect to  $\tau$  yields,

$$[1 + n_t - k_{t+1}^2 \Omega'(k_{t+1}) f''(k_{t+1})] g_\tau^k = -(s_w + \phi'(\tau_t) k_{t+1}) + s_r g_\tau^r \quad (31)$$

Thus,

$$g_\tau^k = \frac{s_r g_\tau^r - (s_w + \phi'(\tau_t) k_{t+1})}{\Delta} \quad (32)$$

On the other hand, from (23),

$$g_\tau^r = f''(k_{t+1}) g_\tau^k \quad (33)$$

Solving this system for  $g_\tau^r$ ,

$$g_\tau^r = \frac{-(s_w + \phi'(\tau_t) k_{t+1}) f''(k_{t+1})}{\Delta - f''(k_{t+1}) s_r} \geq 0 \quad (34)$$

which proves (i). The inequality in (ii) follows immediately because, from (33),  $\text{sign } g_\tau^r = -\text{sign } g_\tau^k$ .

Proceeding in a similar fashion, (21) and (23) lead us to the system:

$$g_k^k = \frac{s_r g_k^r - s_w k_t f''(k_t)}{\Delta} \quad (35)$$

$$g_k^r = f''(k_{t+1}) g_k^k \quad (36)$$

which yields

$$g_k^r = \frac{-s_w k_t f''(k_t) f''(k_{t+1})}{\Delta - f''(k_{t+1}) s_r} \leq 0 \quad (37)$$

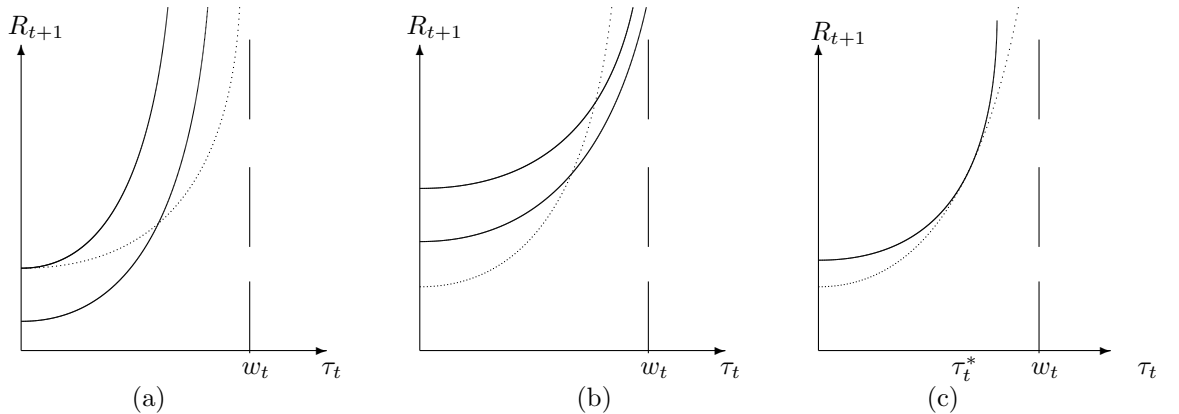
This proves (iii), while (iv) is then implied by (36). ■

Now we will use (12) and (30) to restate the agency's problem as:

$$\begin{aligned} \max_{\tau_t} \quad & V(\omega(k_t) - \tau_t, R_{t+1}) \\ \text{s.t.} \quad & (i) \quad R_{t+1} = g^r(k_t, \tau_t) \\ & (ii) \quad k_t \text{ given} \end{aligned} \quad (\mathbf{A})$$

A graphical representation of this problem might be helpful. Thus, in Figure 3, we depict the typical shape of the constraint set and the indifference curves for the objective function, given a fixed value  $k_t$ . The positive slope of the (dotted) constraint curve follows from a previous lemma, where we showed that  $\partial g^r / \partial \tau_t \geq 0$ ; its asymptotic behavior when  $\tau$  approaches  $\omega(k_t)$  follows from the Inada condition. Similarly, the shape of the indifference curves can be derived from the assumptions in the utility function. In panel (a), we have represented

Figure 3: The Agency Problem



an extreme case in which the optimal policy is to set  $\tau_t = 0$ , so that the agency is not operative. This situation arises when the wage motivation for immigrants,  $\Omega(w_{t+1})$  is large, so that subsidies from the agency are not necessary. In panel (b), no interior optimal  $\tau_t$  exists, as the agency tries to allocate all wage resources to enhance the arrival of immigrants. This situation, of course, can be ruled out for realistic applications. Lastly, in panel (c) we represent the most interesting case in which there is indeed an interior solution. Our analysis will be confined to this case because, otherwise, any study of the agencies would be superfluous in the first place.

The necessary condition for an interior optimum can be written as:

$$V_\omega(\omega(k) - \tau, g^r(k, \tau)) = V_\tau(\omega(k) - \tau, g^r(k, \tau)) g_\tau^r(k, \tau) \quad (38)$$

where the subscripts  $\omega, \tau$  and  $r$  denote partial derivatives. The interpretation is straightforward. The left-hand side of this expression describes the marginal effect of a change of  $\tau_t$  on the utility of the agent during her first period of life; the RHS corresponds to the marginal effect on the utility of the agent at her second period of life. In an optimal policy, both marginal effects should be equal. This is analogous to the usual Euler equation  $u'(c_{1t}) = \beta R_{t+1} u'(c_{2t+1})$ , but here the agency takes into account the effects of its decisions on the process of capital accumulation and future factor prices. It is this tradeoff of the agency that Figure 3(c) illustrates.

## 2.5 Equilibrium Dynamics

Given our assumptions, we have an optimal action for the agency for any given initial value  $k_t$ . Let us write such an optimum as

$$\tau_t = \tau(k_t) \quad (39)$$

It is now possible to introduce (39) into (29), in order to obtain the kinematic equation

$$\begin{aligned} k_{t+1} &= g^k(k_t, \tau(k_t)) \\ &= \kappa(k_t) \end{aligned} \quad (40)$$

This equation, jointly with the population rule

$$\begin{aligned} P_{t+1} &= P_t (1 + \phi(\tau(k_t)) + \Omega(\omega(\kappa(k_t)))) \\ &= \chi(k_t, P_t), \end{aligned} \quad (41)$$

describes the dynamics of capital per capita and total population,  $\{k_t, P_t\}$ , over time.

**Definition 1** *A Perfect Foresight Competitive Equilibrium for the basic immigration model is a sequence  $\{k_t, P_t, w_t, R_t, \tau_t\}$  such that, for every  $t$ ,*

- (i)  $w_t = \omega(k_t)$  and  $R_t = r(k_t)$
- (ii) Given  $(k_t)$ ,  $\tau_t$  solves the agency problem **(A)**,
- (iii) Given  $\{w_t, \tau_t, R_{t+1}\}$ ,  $s_t$  is the optimal savings function in (6), and
- (iv) The kinematic equations (40) – (41) are satisfied. ■

The following results establish necessary and sufficient conditions for the existence of a non-trivial steady-state for the process of capital accumulation. They are the counterpart in the immigration model to sufficient and necessary theorems in the conventional OLG model.<sup>17</sup>

**Proposition 3** *Consider the system (40) – (41), which describes the evolution of  $\{k_s, P_s\}$  over time. This system has a non-trivial steady state if (i)  $s_\tau > 0$ ; (ii)  $\lim_{k \rightarrow 0} g_k^k + g_\tau^k \frac{d\tau}{dk} > 1$ ; and (iii)  $\lim_{k \rightarrow \infty} f'(k_t) = 0$ .*

**Proof.** The proof is immediate. By Proposition 2, the first condition ensures that the perfect foresight equilibrium is well defined. From (29),  $\frac{dk_{t+1}}{dk_t} = \kappa'(k_t) = g_k^k + g_\tau^k \frac{d\tau}{dk}$ ; so the second condition ensures that the graph of  $k_{t+1}$  against  $k_t$  is above the 45-degree line for  $k_t$  sufficiently small, while the third implies it is below for large  $k_t$ . This concludes the proof. ■

**Proposition 4** *If*

$$\begin{aligned} \lim_{k \rightarrow 0} \frac{\omega'(k)}{1 + \Omega(0)} &< 1, \text{ and} \\ \frac{\omega'(k)}{1 + \Omega(0)} &< 1 \text{ for all } k > 0, \end{aligned}$$

*then the (trivial)  $k^* = 0$  is the unique steady state for the immigration model.*

<sup>17</sup> See, for example, Gador and Ryder (1986).

**Proof.** Suppose  $\tau_t = e_t = 0$  and  $s_t = w_t$ . Clearly, if the result holds for those conditions, it also holds when  $s_t < w_t$  and  $\tau_t > 0$ . Now, (21) becomes:

$$k_{t+1} = \frac{\omega(k_t)}{1 + \Omega(\omega(k_{t+1}))} \quad (42)$$

It follows by differentiation that:

$$\frac{dk_{t+1}}{dk_t} = \frac{\omega'(k_t)}{1 + \Omega(\omega(k_{t+1})) - k_{t+1}^2 \Omega'(\omega(k_{t+1})) f''(k_{t+1})} \quad (43)$$

When  $k_t = 0$ , so is  $k_{t+1}$ . Hence, at  $k_t = 0$ ,

$$\frac{dk_{t+1}}{dk_t} = \frac{\omega'(k_t)}{1 + \Omega(0)} \quad (44)$$

If this is smaller than 1 and it continues to be so for  $k_t > 0$ , it is clear that  $k_{t+1} < k_t$  for all  $t$ . Then the result follows. ■

The sign and magnitude of  $\tau'(k)$  has also implications for the dynamics of  $k_t$  around a positive steady-state, whenever such a steady-state exists. In effect, recall that in standard overlapping generations models, the assumption  $s_r > 0$  implies a positive slope for the locus  $k_{t+1} = \kappa(k_t)$ , and then it rules out the possibility of cyclical convergence to the steady state. That is not the case when the immigration elements are taken into account. In our case, it follows from Eq. (40) that  $\kappa'(k) > 0$  iff  $g_k^k(k) > g_\tau^k(k) \tau'(k)$ , where  $g_k^k(k) > 0$  and  $g_\tau^k(k) < 0$ . Thus, if  $\tau'(k)$  is positive and large near the steady-state,  $\kappa'(k) < 0$  and the system will tend to exhibit cycles. The implicit chain of causality can be described as follows. Given an increase in  $k_t$ , the positive  $s_r$  tends to increase  $k_{t+1}$ , as in the standard model. However, if  $\tau'(k_t) > 0$ , the increase in  $k_t$  also increases the effort of the current generation to attract new immigrants, which tends to decrease  $k_{t+1}$ . The outcome will be the result of two opposite forces.

It turns out that the sign of  $\tau'(k)$  is a complicated function of the production and savings functions. To see that, consider  $k_t = k$  as fixed, and write the agency's problem as

$$\begin{aligned} \max_{\tau} \quad & u(c_1(k, \tau)) + \beta u(w(k) - \tau - c_1(k, \tau)) \\ \text{s.t.} \quad & R = g^r(k, \tau) \end{aligned} \quad (45)$$

where  $c_1(k, \tau)$  is the solution of the consumer's problem for a given  $\tau$ .

Using the envelope Theorem, the first order condition can be written as  $R = s g_\tau^r$ . Now, differentiation of both sides yields:

$$\frac{d\tau}{dk} = \frac{g_\tau^r(-s_w k f'' + s_r g_k^r) - g_k^r}{(g_\tau^r - s g_{\tau\tau}^r - g_\tau^r(s_r g_\tau^r - s_w))} + \frac{s g_{\tau k}^r}{(g_\tau^r - s g_{\tau\tau}^r - g_\tau^r(s_r g_\tau^r - s_w))} \quad (46)$$

From the second order condition for the problem above, the denominator in this expression is positive. The numerator in the first term is a substitution

effect, and is always positive, as it can be verified.<sup>18</sup> The numerator in the second term is an income effect, and its sign depends on  $g_{\tau k}^r$ . Using those facts, and appealing to the equality in (35), we obtain

$$\frac{d\tau}{dk} \propto (g_{\tau}^r g_k^k - g_k^r) + s g_{\tau k}^r \quad (47)$$

Therefore, the possibility of cycles cannot be ruled out just based on the sensibility of the savings with respect to interest rate. Nevertheless, simulations reported below lead us to believe that, for reasonable representation of the economy of the United States,  $\tau'(k)$  is certainly positive but relatively small, so that the convergence to the steady-state does not involve cycles.

### 3 Welfare

In principle, the presence of the agencies has important welfare implications. First, they ensure that the life-time utility of an agent is increasing on level of *per capita* capital of her generation.<sup>19</sup> This is proved through the following proposition, where we show that the value function corresponding to the  $t$ -agency's problem is increasing in  $k_t$ .

**Proposition 5** *Let  $W(k)$  denote the value function for the agency problem. That is,*

$$W(k) \equiv V(\omega(k) - \tau^*(k), R_{t+1}) \quad (48)$$

where

$$\begin{aligned} \tau^*(k) = \arg \max_{\tau} & V(\omega(k) - \tau, R_{t+1}) \\ \text{s.t.} & R_{t+1} = g^r(k_t, \tau_t) \end{aligned} \quad (49)$$

Then,

$$W'(k_t) \geq 0.$$

**Proof.** Introducing the constraint into the objective, and differentiating with respect to  $k_t$ , it results:

$$\frac{dW(k_t)}{dk_t} = -V_w k_t f''(k_{t+1}) + V_r g_k^r + (-V_w + V_r g_{\tau}^r) \tau'(k_t) \quad (50)$$

where, as usual, we omit the arguments of some functions. From the first order condition of the agency's problem —see Eq. (38)—, the last term vanishes, and one obtains:

<sup>18</sup> This can be easily seen by recalling, from eq. (35), that  $-s_w k f'' + s_r g_k^r = g_k^k > 0$ . Now, since  $g_k^r > 0$  and  $g_k^r < 0$ , the assertion is obviously true.

<sup>19</sup> This does *not* rule out the possibility of (socially) inefficient overaccumulation, which is a different issue.

$$W'(k_t) = -V_w k_t f''(k_t) + V_r g_k^r, \quad (51)$$

as implied by the Envelope Theorem. The first term in the RHS is positive, while the second one is negative. Further, at an interior optimum,  $\frac{V_w}{V_r} = g_\tau^r$ . Thus, to show that  $W'(k_t) \geq 0$ , it suffices to show that

$$g_k^r \geq g_\tau^r k_t f''(k_t)$$

Substituting from (33) and (36), some trivial algebraisms show that

$$g_k^r - g_\tau^r k_t f''(k_t) = k_{t+1} k_t$$

which is obviously positive. This ends the proof. ■

An alternative heuristic argument might be helpful here. Let  $(\tau(k), e(k))$  be the optimal action of the agency when  $k_t = k$  and consider a pair  $(k', k)$  such that  $k' > k$ . Given  $k'$ , a possible action of the agency is: first, to select  $\tau(k')$  so that the disposable income remains unchanged. That is,

$$\omega(k') -$$

$\tau(k') = \omega(k) - \tau(k)$ ; (52) next, to use the additional proceeds to finance a greater level of immigration effort, so that  $e'(k') - e(k) = \tau(k') - \tau(k) = \omega(k') - \omega(k)$ . In a preceding result, it was shown that

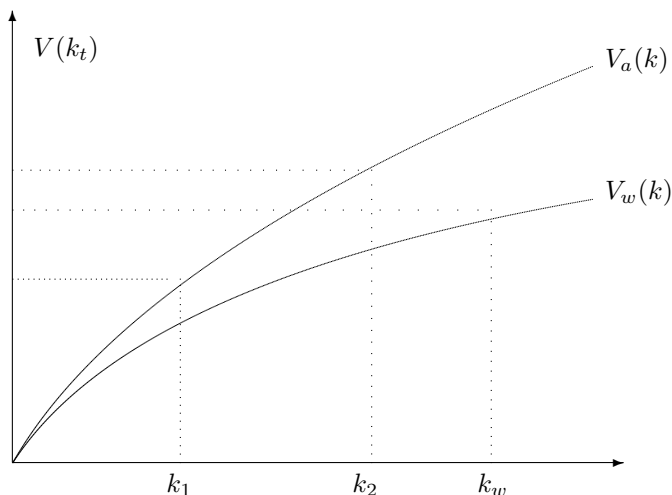
$$g_\tau^r \equiv \frac{\partial R_{t+1}}{\partial \tau} \geq 0,$$

and it is possible to show that the assertion remains valid even if  $w - \tau$  is kept fixed. Then, under the proposed policy, the increase in *per capita* capital enlarges the set of consumption possibilities of the typical agent. Since this is a feasible policy, the proposition should hold.

Thus, from the standpoint of each generation, its agency is welfare-improving, and each individual is better off when an agency is controlling the immigration process on his behalf. It turns out, however, that the operation of the agencies decreases the speed of capital accumulation in the economy. Hence, the benefit of a current generation is partially paid for future generations, which are forced to begin with a smaller level of capital than otherwise. Moreover, it can be readily verified that an economy with operative agencies converges to a steady-state level of *per capita* capital smaller than an identical economy in which no agency operates.

This motivates a natural question: consider an individual who lives at the steady-state of this economy. Is this individual worsened or improved by the continuous operation of the agencies and their active control of immigration? The answer depends on the magnitude of the gap between the steady-state levels of capital with and without agencies. In Figure 4, we represent two different

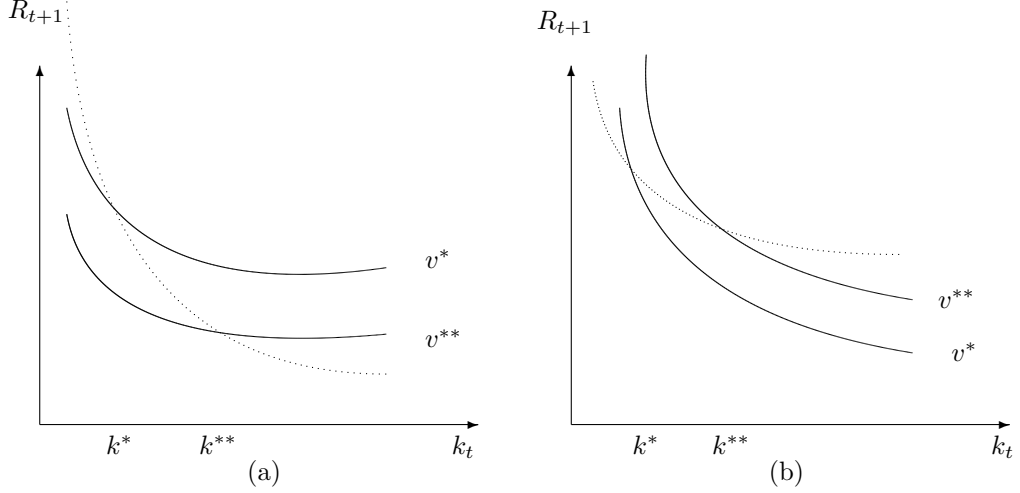
Figure 4: Welfare Effects of Immigration Promotion



situations. The level  $k_w$  denotes the steady-state level of (*per capita*) capital of the economy without agencies and immigration. The levels  $k_1$  and  $k_2$  represent two possible steady-states for the economy with agencies. In turn, the functions  $V_a$  and  $V_w$  represent the lifetime utility of an agent born in a generation with a *per capita* level of capital  $k_t$ , with and without agencies, respectively. If the economy with agencies converges to  $k_2$ , the gap with respect to  $k_w$  is relatively small, so that individuals born at the steady-state are actually better off with the agencies. If the operation of the agencies leads the economy to  $k_1$ , the opposite is true: the gap with respect to  $k_w$  is significantly large, and the individuals who live at steady-state would be better off had the promotion of immigration been historically prohibited. Of course, such a prohibition would not be sustainable, because each generation might have an incentive to improve its own welfare by offering subsidy to immigrants, whatever the previous generations have already done, and whatever the implications for the long future might be. The situation represents a clear example of intergenerational externality in which some type of social contract would be convenient to eliminate a social inefficiency, but the finite lives of individuals makes any intergenerational contract difficult to achieve.

Incidentally, the result that individuals' welfare is increasing in *per capita* capital does not hold when the agencies are not in operation. In other words, without an immigration agency, an increase in the level of  $k_t$  is *not* necessarily welfare-improving for the agents of generation  $t$ . The logic is that, the increase in  $w_t$  associated with the increase in  $k_t$  also induces additional savings, which tend to reduce the future interest rate,  $R_{t+1}$ . If the reduction in the interest rate is not very large, the utility of the representative agent increases. This is illustrated in Figure 5(b), where the increase from  $k^*$  to  $k^{**}$  leads to a movement

Figure 5: Welfare Effects of Capital Intensity without Agency



from the indifference curve  $v^*$  to  $v^{**}$ , with  $v^{**} > v^*$ . On the other hand, when the reduction in the interest rate is very large, the average utility might be reduced. This is illustrated in Figure 5(a), where the increase from  $k^*$  to  $k^{**}$  leads to a movement from the indifference curve  $v^*$  to  $v^{**}$ , with  $v^* > v^{**}$ . For completeness, we derive the conditions under which the latter situation arises.

First, set  $\tau \equiv 0$  and write the indirect utility function as  $V(k, R) = \mathcal{U}(c_1(k_t, R_{t+1}), (w(k_t) - c_1(k_t, R_{t+1})) R_{t+1})$ . Since  $R_{t+1}$  and  $k_t$  are linked by  $R_{t+1} = g^r(k_t)$ , we can also write  $V(k, R) = \mathcal{U}(c_1(k, g^r(k)), (w(k) - c_1(k, g^r(k))) g^r(k))$ , where the time-subscript  $t$  has been omitted. Hence,

$$\begin{aligned} \frac{dV}{dk} = & (\mathcal{U}_1 - \mathcal{U}_2 R) \frac{dc_1}{dk} + (\mathcal{U}_1 - \mathcal{U}_2 R) g_k^r \frac{dc_1}{dR} \\ & + \mathcal{U}_2 s g_k^r + \mathcal{U}_2 R \omega'(k) \end{aligned} \quad (53)$$

where  $\mathcal{U}_i \equiv \frac{\partial \mathcal{U}}{\partial c_i}$ ,  $i = 1, 2$ , and where we have omitted the arguments of some functions.

From the first order condition in (4), this collapses to:

$$\frac{dV}{dk} = \mathcal{U}_2 s g_k^r + \mathcal{U}_2 R \omega'(k) \propto g_k^r s - R k f'' \quad (54)$$

Clearly,  $\frac{dV}{dk} < 0$  iff  $g_k^r s < R k f''(k)$ . From the corresponding version of (21),  $g_k^r = \frac{-s_w k f''(k_t) f''(k_{t+1})}{1 - s_r f''(k_{t+1})}$ . Substituting, it follows that  $\frac{dV}{dk} < 0$  iff

$$\frac{-s s_w}{1 - s_r f''(k_{t+1})} < \frac{f'(k_{t+1})}{f''(k_{t+1})} + \frac{1}{f''(k_{t+1})} \quad (55)$$

It is easy to find situations in which this condition holds.<sup>20</sup> Nevertheless, our experience suggests that  $V'(k_t) \geq 0$  for most common situations. That is why, in Figure 4, we represented both value functions —with and without the agency— as increasing functions of  $k_t$ .

## 4 A Two-country Immigration Game

The previous analysis focused on the dynamics of a *host* economy, ignoring both the effects of that dynamics on the *source* economy, and the reactions that it might induce. Moreover, the analysis implicitly assumed that the source economy did *not* have any immigration agency, so that the migrants flow basically depended on the host economy's policies. That sort of asymmetry was useful to point out our basic ideas, but now it is convenient to allow for a greater interaction between both economies.

To that end, let us consider two countries,  $i = 1, 2$ , each one similar to the economy described earlier, except by the features to be introduced here. In particular, both countries have identical demographic structure: individuals live for two periods, working in the first and retiring in the second. Individuals can move from a country into the other, depending on the wages differential and the subsidies offered by the agencies from both countries.<sup>21</sup> In order to focus our attention on population movements, let us continue to assume that capital is country-specific, so it cannot freely move from one country to other. In short, we shall now say that

$$\frac{M_{t+1}^1}{P_t^1} = \Omega(w_{t+1}^1 - w_{t+1}^2) + \phi(e_t^1 - e_t^2), \quad (56)$$

and

$$\frac{M_{t+1}^2}{P_t^2} = -\Omega(w_{t+1}^1 - w_{t+1}^2) - \phi(e_t^1 - e_t^2), \quad (57)$$

where  $\Omega(0) = \phi(0) = 0$ ,  $\Omega' > 0$ , and  $\phi' > 0$ . Thus, when the wages in country 1 are greater than in country 2, people tend to move from the latter country into the former. The same is true when the incentives provided by country 1's

<sup>20</sup> For example, the inequality in (55) is satisfied for the utility and production functions  $u(c) = \ln c$ ,  $f(k) = -\frac{1}{2}k^2 + Bk$ ,  $k < B$ , whenever  $k$  is close to  $B$  and  $B$  is sufficiently large. In this case, the condition becomes

$$\frac{s s_w}{1+n} = \frac{1}{1+n} \left( \frac{\beta}{1+\beta} \right)^2 \omega(k) > B - k + 1,$$

where  $\omega(k) \equiv f(k) - k f'(k)$ . This inequality amounts to  $\frac{1}{2(1+n)} \left( \frac{\beta k}{1+\beta} \right)^2 > B - k + 1$ , which becomes  $B^2 > 2(1+n) \left( \frac{1+\beta}{\beta} k \right)^2$  when  $k \simeq B$ . Clearly, the result must hold for  $B$  large enough. It can be readily seen that the fact that  $f(k)$  has a blisspoint is immaterial for the result.

<sup>21</sup> Thus, we continue to ignore the influence of interest rates.

agency are greater than those provided by the agency in country 2. Of course, the statements also hold in reversed order.<sup>22</sup>

Proceeding as before, we obtain the set of equations:

$$\begin{aligned} k_{t+1}^i &= \frac{S_t^i}{P_{t+1}^i} \\ &= \frac{P_t^i s_t^i(w_t^i - \tau_t^i, R_{t+1}^i)}{P_t^i(1 \pm \phi(e_t^1 - e_t^2) \pm \Omega(w_{t+1}^1 - w_{t+1}^2))}, \quad i = 1, 2 \end{aligned} \quad (58)$$

Given fixed values for  $k_t^1$ ,  $k_t^2$ ,  $\tau_t^j$ , and  $e_t^j$ , the problem of the immigration agency in country  $i$  is to solve:

$$\begin{aligned} \max_{e_t^i, \tau_t^i} \quad & V^i(w_t^i - \tau_t^i, R_{t+1}^i) \\ \text{s.t.} \quad & (i) \quad \tau_t^i = |e_t^i| \leq w_t^i \\ & (ii) \quad w_t^i = f(k_t^i) - k_t^i f'(k_t^i) \\ & (iii) \quad R_{t+1}^i = f'(k_{t+1}^i) + 1 \\ & (iv) \quad w_{t+1}^i = f(k_{t+1}^i) - k_{t+1}^i f'(k_{t+1}^i) \\ & (v) \quad k_t^i, k_t^j, \tau_t^j, e_t^j \text{ given,} \end{aligned} \quad (59)$$

where  $k_{t+1}^i$  satisfies the expression above.

Let us now introduce the following notions.

**Definition 2** Given  $k_t^1$  and  $k_t^2$ , an reactive policy function for the agency in country  $i$  is a function,  $\tau^i = \tau^i(\tau^j \mid k_t^i, k_t^j)$ , given the optimal  $\tau^i$  for any given value of  $\tau^j$ ,  $i \neq j$ . ■

**Definition 3** Given  $k_t^1$  and  $k_t^2$ , an agencies period-equilibrium is a pair  $(\tau_t^1, \tau_t^2)$ , such that  $\tau_t^i = \tau^i(\tau^j)$ ,  $i \neq j$ .

To avoid irrelevant technical issues, we will assume that the immigration ‘‘taxes’’ in both countries are restricted to a compact subset of the interval  $[0, w(k_t^i)]$ , so that the reaction functions are surely well-defined. The following result is immediate.

**Proposition 6** Let  $\tau_t^i$ ,  $i = 1, 2$  be given. Then, there is a pair  $(\tau_t^1, \tau_t^2)$  such that  $\tau_t^i = \tau^i(\tau_t^j)$ ,  $i, j = 1, 2$ ,  $i \neq j$ .

**Proof.** By assumption, the choice set is compact for both agencies. Then the result is implied by a small argument around Brower’s theorem. ■

Finally, we introduce the following equilibrium notion.

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<sup>22</sup> The parametrization above preserves a formal similarity with the previous analysis. Strictly speaking, it requires the existence of a ‘‘rest of the world’’ country to equalize the percentage increase of population in one country with the percentage decrease in the other when the countries have different population levels.

**Definition 4** *A Perfect Foresight Competitive Equilibrium for the two agencies game is a sequence  $\{k_t^i, P_t^i, w_t^i, R_t^i, \tau_t^i\}$ ,  $i = 1, 2$ , such that, for every  $t$ ,*

- (i)  $w_t^i = \omega(k_t^i)$  and  $R_t^i = r(k_t^i)$
- (ii) Given  $(k_t^i, k_t^j, \tau_t^j)$ ,  $\tau_t^i$  solves the agency  $i$ 's problem,
- (iii) Given  $\{w_t^i, \tau_t^i, R_{t+1}^i\}$ ,  $s_t^i$  is the optimal savings function in (6),
- (iv)  $\tau_t^1$  and  $\tau_t^2$  are an agencies-period-equilibrium;
- (v) The kinematic equation (58) is satisfied, and the populations evolve in accordance to the denominators in that expression. ■

A word of warning seems to be appropriate. It is not difficult to see that, in any equilibrium, both countries should be interested in preserving their respective populations; that is,  $\tau_t^1 \geq 0$  and  $\tau_t^2 \leq 0$ . This seems to contradict ordinary experience, according to which countries with low level of capital apparently encourage the exit of migrants toward more developed countries. The reason for this, of course, is that those migrants preserve close ties with their original countries, to which they send valuable economic resources.<sup>23</sup> Nevertheless, a negative  $e_t$  in our model can be interpreted as a willingness of the agency to make non-market transfers to immigrants or *native-born* working population, because of its positive effects as a complementary factor for native capital. Since that is something that most governments certainly do, the implication of our model is not really contradicted by the common experience.

The techniques in previous sections can be used to analyze this equilibrium. For our purposes, the most relevant question would be whether the equilibrium is unique and stable, in the sense that the *per capita* level of capital tends to equalize in both countries. Since the question of uniqueness and stability does not have very conclusive answers even in standard OLG models, there is not hope to obtain more precise results in the current, more complex, setting. In principle, we could have a vicious situation in which people flow from the most populated country into the less populated one, increasing an initial difference in per capita capital. Such a situation is not likely to be empirically relevant. Thus, rather to pursuit in that direction, we will now move toward very suggestive simulations.

## 5 Quantitative Analysis

The model outlined above was purposely stylized, and its empirical application would require several intermediate stages. A first step would be to determine a concrete counterpart for the abstract concept of “agency.” Similarly, the empirical equivalent of a “period” should be specified, substituting the 2–period assumption for a more realistic description of individuals’ lives. More importantly, functional forms and parameters values for the immigration functions of

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<sup>23</sup> The introduction of those twists into our model leads to conceptually significant problems. For instance, when migrants were assumed to keep nexus with their native countries, it is extremely hard to determine whom utility each agency maximizes. A vivid discussion of legal and ethical aspects of this problem can be found in Chang (1996).

the economy in point should be obtained. Since the framework of the model is new, some of the relevant functions have not been estimated yet, and so they should be inferred from raw data.

Here we will use instead an alternative, admittedly more naive approach, to assess the ability of the model to describe the dynamics of immigration in the United States. Following Jones and Manuelli (1992), we will interpret the time unit of the model as a period of around 30 years. Then we will specify the production and utility functions, taking appropriate transformations of the yearly-based estimates of the relevant parameters. Finally, we will represent the immigration function in the simplest way consistent with the assumptions and underlying intuition of the model, and we will ask ourselves the following question: given the parameters in the other functions, can we find parameters values for the immigration functions such that the resulting economy roughly replicates a significant set of relevant observable variables in U.S.?

The answer to that question seems to be positive. While this is perhaps a weak test, it reinforces our beliefs that, under more realistic calibration, the model should explain important features of modern immigration processes and the political economy behind them. In the process, it will provide a back-of-the-envelope estimate of the effects of subsidies to immigration on capital accumulation and welfare of the above mentioned economy.

In our simulations, the utility and production functions adopt standard forms:

$$f(k_t) = A (1 + g)^t k_t^\alpha, \quad (60)$$

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1 - \sigma} \quad (61)$$

where productivity is assumed to increase at the exogenous rate  $g$ . We will also allowed for capital depreciation at the rate  $\delta$ .<sup>24</sup> Further, we will parametrize  $\Omega$  and  $\phi$  in such a way that these conditions hold: (i)  $\Omega(0) = \phi(0) = 0$ , (ii)  $\Omega' > 0$ ,  $\phi' > 0$ , (iii)  $\Omega'' < 0$ ,  $\phi'' < 0$ , and, for any  $m$ , (iv)  $\phi(m) = -\phi(-m)$ . Functions satisfying this are:

$$\Omega(w) = P w^{1/2} + F \quad (62)$$

$$\phi(e) = \gamma B |e|^{1/2} \quad (63)$$

where  $\gamma$  is a dummy parameter which equals 1 when  $e \geq 0$ , and  $-1$  otherwise.

In Table 1.1, we present the selected values for the parameters  $A$ ,  $\alpha$ ,  $\delta$ , and  $g$ . Those values are simply transformations of commonly reported estimates for 1-year periods. We also select  $F$  to roughly match the rate of growth of the native population of the United States; that is, the population growth excluding the part due to the entry of immigrants. Further, since we would like to measure the effects of the substitution coefficient of the utility function, we will simulate

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<sup>24</sup> Although those features were not present in previous sections, their inclusion is immediate. If the variables are taken in "intensive-form", the right hand side of Eq. (21) should be divided by  $(1 + g)$  and  $R_{t+1} = f'(k_{t+1}) + 1 - \delta$ . The remaining expressions are basically unchanged.

the model for alternative values of  $\sigma$ , including values above and below 1. This leaves two free parameters,  $B$  and  $P$ , and we propose to select them to satisfy, if possible, the following requirements at the steady-state: (a) the rate of population growth should be around 65%, which corresponds to the rate of growth for US in a 30-years base; (b) the immigration contributions to the agency should represent a “small” fraction of national income —say, for instance, less than 1.5 %— to be consistent with most common assessments of the fiscal burden from subsidies to immigrants; and (c) the level of *per capita* capital,  $k^*$ , should be between 9 and 12.<sup>25</sup>

**Table 1.1**  
*Parameters Values*

<i>Param.</i>	<i>Value</i>
$A$	0.89 x 30
$\alpha$	0.36
$\delta$	$1.04^{30} - 1$
$g$	$1.012^{30} - 1$
$F$	0.52

We summarize, in Table 1.2, the steady-state values emerging from different combinations of  $\sigma$  and  $(B, P)$ . In general, the steady-state values of capital,  $k^*$ , population growth,  $\hat{n}$ , and incidence of immigration cost,  $\tau^*/y^*$ , increase with  $\sigma$ . As a result, the figures seems to be reasonable for  $\sigma = 0.8$  and  $1.5$ , while they seem to be somehow inflated for high values of  $\sigma$ , as  $\sigma = 2$ . It is interesting to notice that for small values of  $\sigma$ , the incidence of immigration cost, i.e., the ratio  $\tau^*/y^*$ , decreases when  $B$  decreases. For higher  $\sigma$ 's, to the contrary, a decrease in  $B$  actually increases the steady-state value  $\tau^*/y^*$ . A natural interpretation is that the elasticity of the incidence of immigrant-supportive expenditure with respect to the price of immigrants, represented by the inverse of  $B$ , is critically affected by the curvature of the utility function.

Our simulations also show a positive link between the tax burden,  $\tau/y$ , and the level of *per capita* capital. That positive link is not as immediate as it might appear at first sight, because the deepening of capital has indeed two opposite effects. On one hand, it makes the economy richer and it increases the marginal productivity of prospective immigrants, increasing both the benefit from attracting working population and the ability to finance such a policy through an increase of  $\tau$ ; on the other hand, since wages increase with higher *per capita* capital, a greater number of immigrants tends to arrive by themselves, so that to provide them with incentives is less necessary than earlier. The simulations

<sup>25</sup> The ratio  $k^*/f(k^*)$  is usually considered to be between 3 and 5. To translate this into a 30-period basis, we take a value 4.5 divided by 30. Given the production function above, that corresponds to a value of  $k^*$  around 9.

simply show that the first effect is stronger for the range of parameters considered here, so that in all cases simulated we found that the pace of  $\tau/y$  was monotonically increasing over time. A neat succinct way to put this is to say that, for economies near the simulated economies, the resources spent to support immigrants would tend to increase with the wealth of the host country: richer generations will find it optimal to dedicate a greater part of their output to enhance immigration, because of the future benefit that immigrants represent. The sensitivity of this nexus depends on the value of  $\sigma$ .

**Table 1.2**  
*Steady-state Values under Agency's Policy*

$\sigma$ ( $B, P$ )	0.8	1.5	2
(0.055, 0.015)	$k^* = 9.94$ $\hat{n} = 67.08\%$ $\frac{\tau}{y} = 0.5\%$	$k^* = 11.08$ $\hat{n} = 70.2\%$ $\frac{\tau}{y} = 2\%$	$k^* = 11.96$ $\hat{n} = 70.5\%$ $\frac{\tau}{y} = 2.1\%$
(0.045, 0.015)	$k^* = 10.04$ $\hat{n} = 66.08\%$ $\frac{\tau}{y} = 0.35\%$	$k^* = 11.51$ $\hat{n} = 68.7\%$ $\frac{\tau}{y} = 1.55\%$	$k^* = 11.26$ $\hat{n} = 72.1\%$ $\frac{\tau}{y} = 3\%$
(0.035, 0.015)	$k^* = 10.12$ $\hat{n} = 65.2\%$ $\frac{\tau}{y} = 0.22\%$	$k^* = 12.00$ $\hat{n} = 68\%$ $\frac{\tau}{y} = 1.2\%$	$k^* = 12.6$ $\hat{n} = 68\%$ $\frac{\tau}{y} = 2.7\%$

**Table 1.3**  
*Steady-state Values without Agencies*

$\sigma = 0.8$	$\sigma = 1.5$	$\sigma = 2$
$k^* = 10.25$ $\hat{n} = 64.01\%$ $\frac{\tau}{y} = 0\%$	$k^* = 13.56$ $\hat{n} = 64.2\%$ $\frac{\tau}{y} = 0\%$	$k^* = 11.96$ $\hat{n} = 70.5\%$ $\frac{\tau}{y} = 0\%$

We now turn to the welfare effects of the control of immigration by part of the agencies. We approach the problem through a comparison with structurally identical economies in which the agencies, however, are *not* allowed to operate. In those benchmark economies, no financial resources are spent to promote immigration, so that population only increases because of internal reproduction

and the arrival of immigrants solely attracted by wages differences. In the United States, this would correspond to the elimination of any welfare benefits for immigrants, the imposition of higher user taxes for the use of public facilities, etc., as proposed by some politicians.

The steady-state values for those comparison economies are reproduced in Table 1.3. A first relevant comparison is the level of utility for a single generation endowed with a given level of capital. This utility is obviously greater for the economies with operative agencies. Thus, in Table 1.4, we present the percentage gain in utility corresponding to a representative set of parameters values  $\sigma$ , using the steady- state value of capital as the reference point. This gain is computed as<sup>26</sup> :

$$100 \cdot \frac{V_a(k_{at}) - V_w(k_{wt})}{V_w(k_{wt})}, \quad (64)$$

where  $V_w$  denotes the utility level without agencies, and  $V_a$  the utility level with agencies. Thus, this formula yields the additional welfare that a single generation which leaves at the steady-state would obtain by adopting an active, agency-controlled, immigration policy. The simulations reveal that this benefit increases with  $\sigma$  and, as one might expect, it decreases when the inverse of the price of marginal immigrants,  $B$ , goes down.

**Table 1.4**  
*t-Generation's Utility Gain at Steady-State*

$\sigma$ ( $B, P$ )	0.8	1.5
(0.055, 0.015)	$k^* = 10.25$ $U_{gain} = 0.20\%$	$k^* = 13.56$ $U_{gain} = 0.25\%$
(0.045, 0.015)	$k^* = 10.25$ $U_{gain} = 0.13\%$	$k^* = 13.56$ $U_{gain} = 0.18\%$
(0.035, 0.015)	$k^* = 10.25$ $U_{gain} = 0.09\%$	$k^* = 13.56$ $U_{gain} = 0.14\%$

<sup>26</sup> The denominator in this expression is always positive in the relevant range.

**Table 1.5**  
*Difference Between Steady-State Capitals (%)*

$\sigma$ ( $B, P$ )	$0.8$	$1.5$
(0.055, 0.015)	$(k_w^* - k_a^*)/k_a^* = 3.1\%$	$(k_w^* - k_a^*)/k_a^* = 18\%$
(0.045, 0.015)	$(k_w^* - k_a^*)/k_a^* = 2.13\%$	$(k_w^* - k_a^*)/k_a^* = 14.8\%$
(0.035, 0.015)	$(k_w^* - k_a^*)/k_a^* = 1.31\%$	$(k_w^* - k_a^*)/k_a^* = 12.0\%$

As discussed earlier, the long-run social drawback of the short-run benefit obtained through immigration-subsidy is a reduction of the the level of capital, and this is evident from Table 1.5. In the typical simulation, we found that for two economies which begin with equal level of capital ( $k = 0.01$ ), the path of *per capita* capital in the economy without agencies is significantly above the corresponding path for the economy with active agencies after a few periods. We also found that the difference between both paths tends to increase with the coefficient of intertemporal substitution,  $\sigma$ . Finally, it can be shown through our simulations that, for the parameters considered here, after a small number of periods the welfare effects for the economy eventually vanishes. The results suggest that, in the long-run, the welfare implications of the operation of the agencies in an economy near the American economy is likely to be almost null.

As a final point, we would like to extend our simulations to the two-country game previously described. The most natural way to proceed would be to consider two interacting countries, 1 and 2, one country corresponding to U.S. and the second representing other relevant country —as, for example, Mexico. A serious limitation for a serious calibration is that several important parameters of the foreign economy are not readily available.

It is still insightful, however, to study the working of the model under some heroic assumption: it will be supposed that both countries are described by the parameters in Table 1.1, and that they only differ for their levels of capital. We shall now explore the *qualitative* behavior of both economies under those conditions. The exposition will be concise, and several details will be omitted.

The following pattern emerges: if the agency (government?) of country 2 is making a huge transfer to its working population to incentive them to *stay* there rather than migrate to country 1, then country 1 will find it optimal not to spend resources to subsidize immigrants, because it is not likely that they come anyway.<sup>27</sup> Likewise, if country 2 makes a huge effort to induce its working population to emigrate, it is also optimal for country 1 not to subsidize immigrants, because country 2 is virtually doing that service anyway.<sup>28</sup> But there is a middle ground in which subsidies to immigrants are convenient. Not

<sup>27</sup> For example, it might not be easy for United States to attract working population from Switzerland.

<sup>28</sup> Such a situation will never occur in an equilibrium.

surprisingly, it is found that for any action of the second country, the optimal  $e_1$  increases with the level of  $k_1$  and, of course, the same result is valid for country 2.

For any given  $k_1$  and  $k_2$ , there is an equilibrium point  $(e_1, e_2)$ . It involves a positive subsidize to immigrants by part of country 1, and a negative effort by part of country 2 — which, again, should be understood as a transfer to its working population in addition to the market-determined wages. This equilibrium determines the next values of capital, say  $k_{1t+1}$  and  $k_{2t+1}$ . Thus, in omitted computations, we have traced the path of the capital in each country, and we have compared those paths with the evolution of capital when agencies do not operate. We found that the level of capital with the agencies is in *both* countries smaller than under a situation in which only the market forces are in operation. This occurs because, in both economies, the current generations are forced to spend resources in their competition for attracting working population. This suggests the existence of potential gain from a cooperative equilibrium in which contemporaneous agencies from both countries agree upon a mutually convenient policy.

## 6 Conclusions

In this paper, we have used an overlapping generations model to study the dynamic interaction of capital accumulation, population growth, and immigration policy in modern economies. In its basic form, the model considers a single economy whose immigration policy at each period reflects the interest of the generation born at that time. This immigration policy is implemented by a collective organization which follows a Ramsey-type program: it selects an immigration policy taking into account individuals' independent reactions to its own decisions. It is then shown that, in the short-run, each generation will have an incentive to promote the entry of immigrants, because they represents an immediate benefit. In the long-run, however, this benefit is countervailed by a decrease in the speed of capital accumulation. In an extension, we also considered an immigration game between two countries, each one with its own agency. In the equilibrium of this game, the immigration policy of each country depends on the immigration policy of the other.

The analytical discussion was then followed by numerical simulations, where we constructed artificial economies which roughly resemble the economy of the United States. The conclusion from numerical exercises can be summarized as follows: to the extent to which our parametrization is close to the United States, it seems that the short-term benefits from subsidizing immigrants in that economy is almost completely offset by the negative effect on the long-run level of *per capita* capital. Put in other words, the existence of migrants-supportive programs is neither highly detrimental nor highly beneficial in the long-run. Curiously enough, however, each individual generation will find it convenient to preserve those programs, because their elimination would represent an immediate, uncompensated loss.

Our results, of course, are subject to obvious limitations, which we appropriately emphasized, and it would be naive to take them as definite statements. Moreover, several qualifying factors should be included in the analysis, before a final word could be obtained with respect to such important problem. From our own perspective, the main value of the model developed here is to provide a framework in which the dynamic welfare implications of immigration can be clearly addressed. The empirical usefulness of the model should certainly be subject to further analysis.

An agenda for future research should include at minimum set of issues. First, a more careful specification of the immigration functions. Second, a richer specification of the life-cycle aspects of individuals. Third, to allowed for some kind of economic contact between immigrants and their original countries. And, finally, to incorporate our model into a world-equilibrium model. A richer specification of the life-cycle of individuals seems to be specially important, because it would lead to a richer structure of the demand for immigrants at any given period. For example, suppose that each individual lives for three periods, working in the first two periods and retiring at the third. Then, at any moment, the economy would have two different groups of people, each one interested in a different immigration path. This would generate a sort of conflict between different groups of the host economy, which seems to correspond to the modern experience of several economies.

Finally, it is important to point out that that, with minor variations, the model outlined above might be used to study the incentive of older generations to invest in the younger population of the economy. For the older population in this model, an immigrants and a native-born younger are identical: both groups represent an input which increases the marginal productivity of capital. Hence, the same logic that leads to a positive optimal subsidy for immigrants, also leads to a positive optimal investment in the education and training of the working population of an economy. Our model can be readily used to obtain quantitative assessments regarding that important issue.

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